Signal Processing

Midterm

Robotics & Automation pathway

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A. Methods

Hardware

For this Midterm exam, a Dell Inspiron 15 7510 laptop was used to conduct the analysis of an unknown signal.

Software

MATLAB R2023b Update 6 (23.0.2.2485118) 64-bit (win 64) – version available 28th December 2023 – was used to run the scripts provided by the examinator Mr. Guillaume Gibert. Also, the Signal Processing toolbox has also been added to properly execute some side functions.

In order to proceed, a ".csv" file has also been given, and contains values to plot and analyse the signal.

By plotting the signal, we can deduce that this one is divided into several parts due to different stationarity sections. When dealing with stationary signals, it becomes easier to analyse their frequency content, predict their behaviour, and apply filters to manipulate them when they are correctly separated. So it would be necessary to conduct different analyses in this case

Moreover, the Nyquist-Shannon theorem states that to accurately sample an analog signal, the sampling rate must be at least twice the signal's highest frequency component. This prevents aliasing, where high-frequency components are misrepresented as lower frequencies.

Consequently, we can deduce that the maximum frequency would be less than 100 Hz theoretically since our sampling frequency is 200 Hz.

Starting from a time domain signal it is necessary to convert it into a frequency domain signal since we need to extract frequencies. This is why we convolute different functions in either the time or frequency domains. When you multiply functions in the time domain and seek their Fourier transform, the convolution theorem enables you to reframe the product in terms of convolution operations in the frequency domain.

Using window functions to modify signals is essential in signal processing. These functions help shape signals by adjusting their edges, improving how we analyse their frequencies. By combining signals with window functions, we can also create effective filters that target specific frequency ranges, which is handy for tasks like noise reduction or isolating certain sounds.

All in all, we should find a window function g(t) defined in [-T/2 T/2] whose Fourier transform is close to a Dirac, but this is impossible so we will approximate them. For this Midterm, a rectangular (for very high frequencies) and a Blackman (for lower frequencies) windows are a good way to proceed, but the usage of Haan windowing or even Hamming are a good compromise between the first 2 windows.

Lastly, it is important to understand the concept of resolution, important factor to be precise enough for determining frequencies. It represents the space between each calculated point. This would result in a more precise answer when this space is very small. The resolution is $= \frac{Sampling\ frequency}{Duration * Sampling\ frequency}$ which is equal to $\frac{1}{Duration}$. Therefore, by increasing the Duration size, we would decrease the calculation space between each point.

To do so, we have 2 possibilities, either recording a longer signal, or adding a list of 0 at the end of the signal values. The first option should be done if the recording conditions allow it, but in our case we should add a 0 padding to make our signal duration longer.

B. Results

On the following Figure 1 we can observe that the signal is divided into 2 parts (2 times 1 second). This is why we will perform 2 analyses, to have the same stationarity almost everywhere on the reduced signal section.

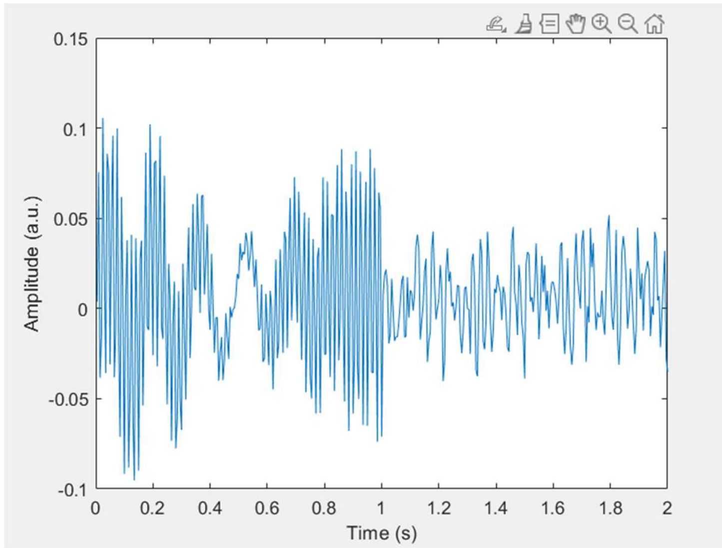

Figure 1 : Plot of the whole original signal

However, by conduction the analysis on the whole signal duration at a time, we get such a graph on Figure 2 that is indicating we certainly have our frequencies at 60Hz and between 5 and 20 Hz (which is our frequency range of interest). But, by studying the whole signal, we could be mixing some components since we do not have the same stationarity everywhere.

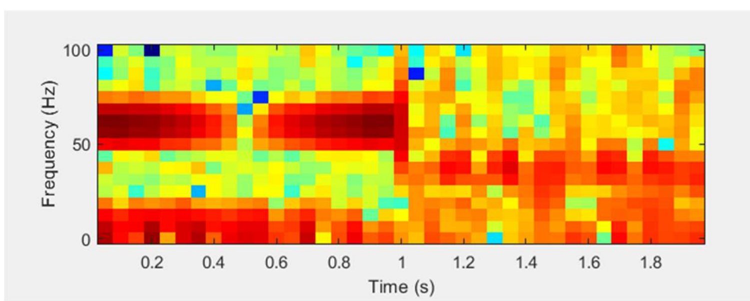

Figure 2 : Plot of the whole original signal

Afterwards, applying windows becomes necessary to go in a depth analysis. So Figure 3, depicts the output after using a rectangular window at 1 Hz whereas Figure 4 is at 0.5 Hz. We can observe that after decreasing the resolution (better calculations) the end of the signal plot over time is 0 in amplitude, since we added 0 to increase the Duration size.

On the central plot, we can see some frequency peaks. However, since the graph is symmetrical we will only consider the left part from 0 to Sampling frequency/2.

From Figure 3 we could read that we are concerned by a peak at 6 and 7 Hz during the first part of the signal (which is the first second).

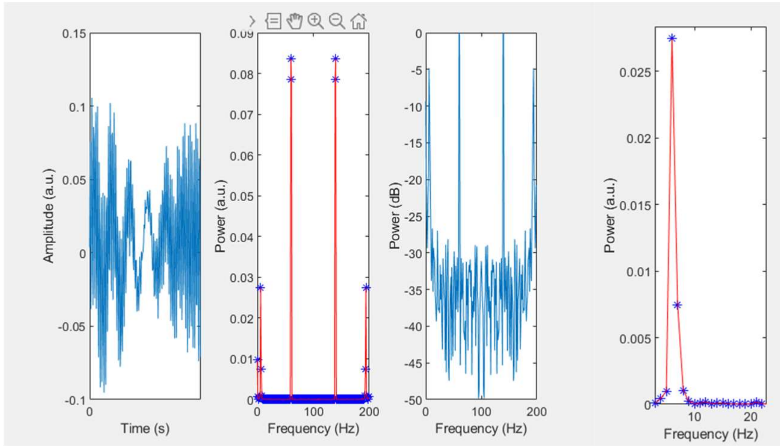

Figure 3 : Plot and zoom during the first second using rectangular window at 1 Hz resolution

However, by having a better resolution for Figure 4 plotting, we are closer to peaks at 6 and 6.5H

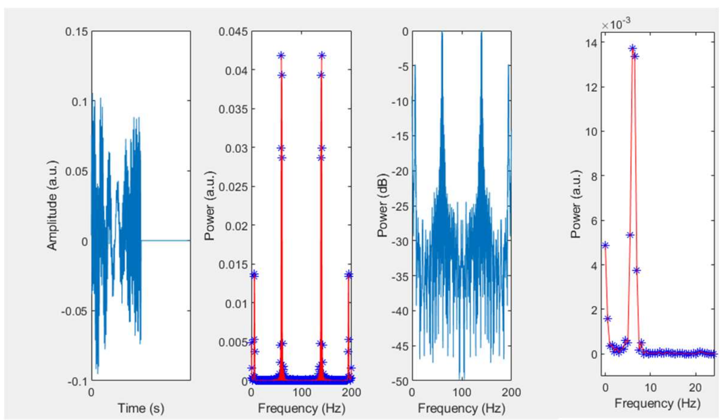

Figure 4 : Plot and zoom during the first second using rectangular window at 0.5 Hz resolution

Now conducting the analysis on the second part of the signal (from 1 to 2 seconds), we are able to see a large peak at 18Hz and a small one at 19Hz. This need a further investigation with a better resolution. We also have a peak at 21Hz but this is out of our range of interest.

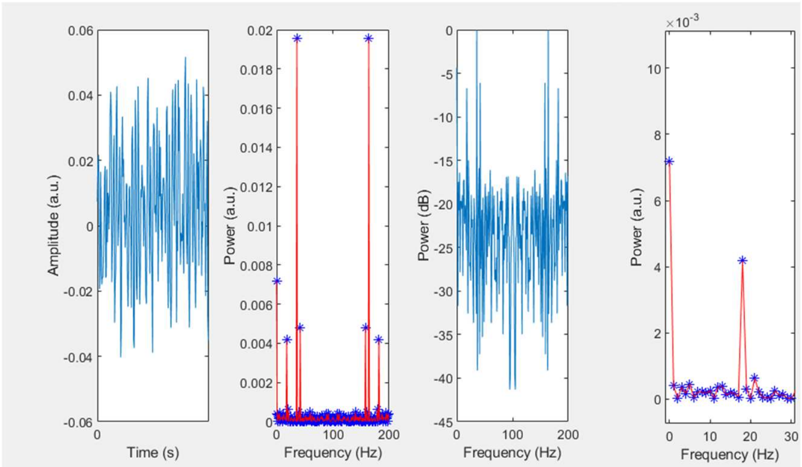

Figure 5 : Plot and zoom during the second second using rectangular window at 1 Hz resolution

Conducting the same principle as before, we have peaks at 18, 18.5Hz but also at 8.5 and 16.5 Hz but the 2 are a bit smaller in Power (a.u.)

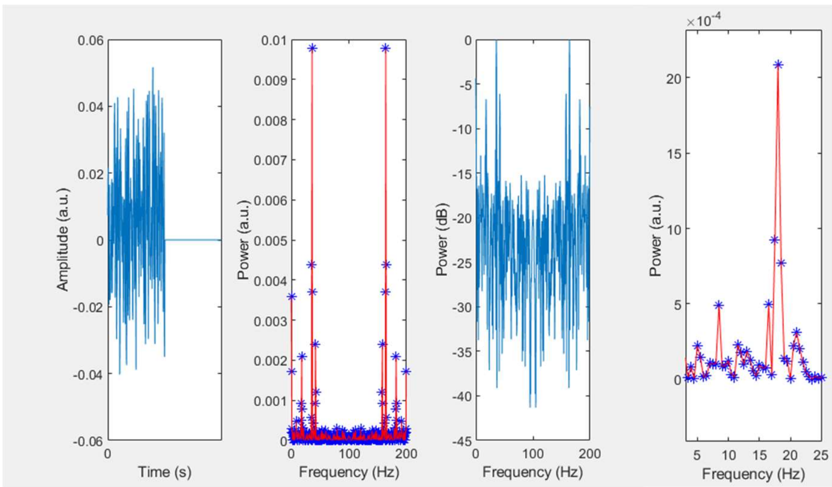

Figure 6 : Plot and zoom during the second second using rectangular window at 0.5 Hz resolution

Knowing that the rectangular window is useful in a very high frequency movement, we can now show the result of a Blackman window at 1 Hz (Figure 7 and 9) and 0.5 Hz (Figure 8 and 10). Again, 0 padding has been used (as explained before) in the second case.

By zooming in the region of interest, we are able to see peaks at 6 and 7 Hz on Figure 7, again we need to improve the resolution to have a more precise frequency result.

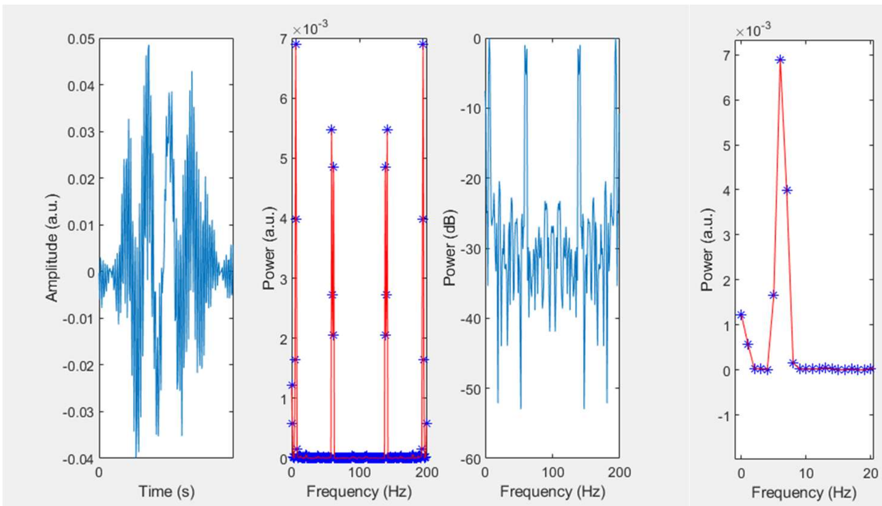

Figure 7 : Plot and zoom of the first second using Blackman window at 1 Hz resolution

Thanks to Figure 8, the resolution at 0.5Hz, we can distinguish the peaks at 6 and 6.5Hz

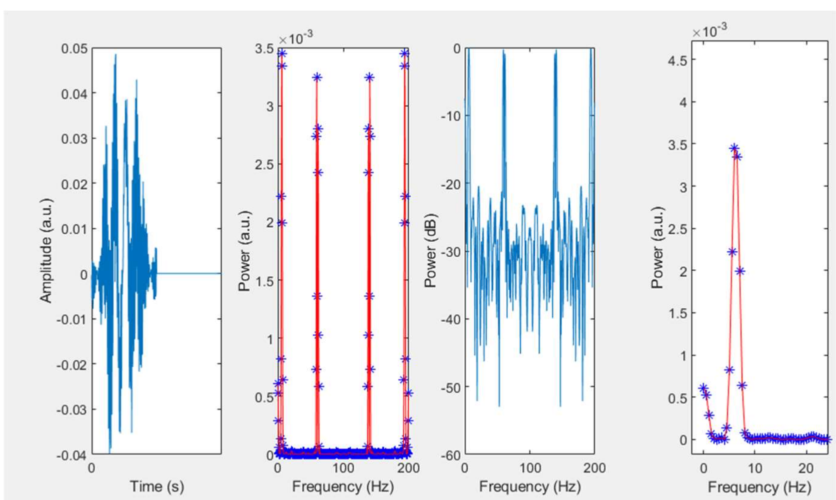

Figure 8: Plot and zoom of the first second using Blackman window at 0.5 Hz resolution

Using the blackman window on the second part of the signal as well, we are barely able to see peaks at 13 -18 and 19Hz

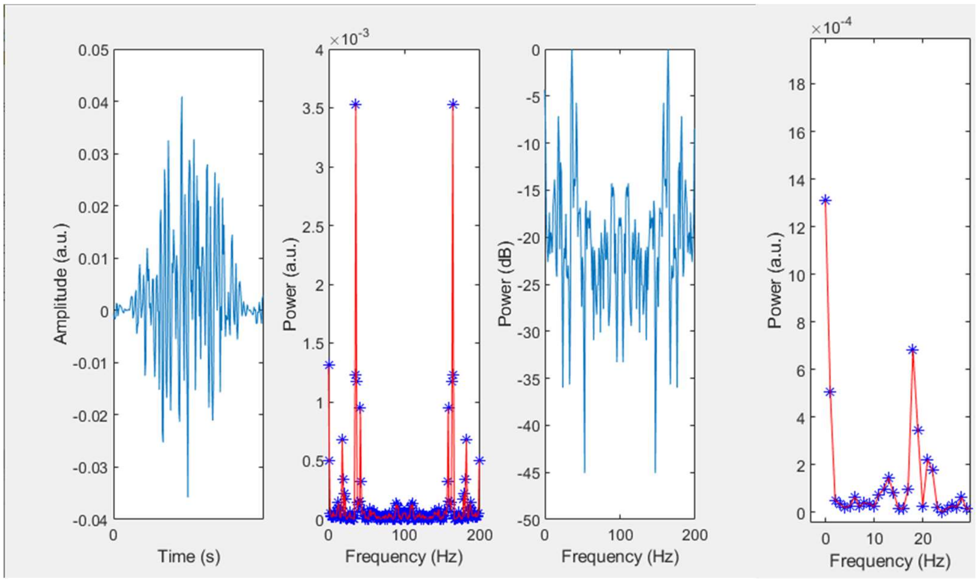

Figure 9: Plot and zoom of the second second using Blackman window at 1Hz resolution

By applying the previous resolution method, we finally have frequencies at 13 – 18 and 18.5 Hz

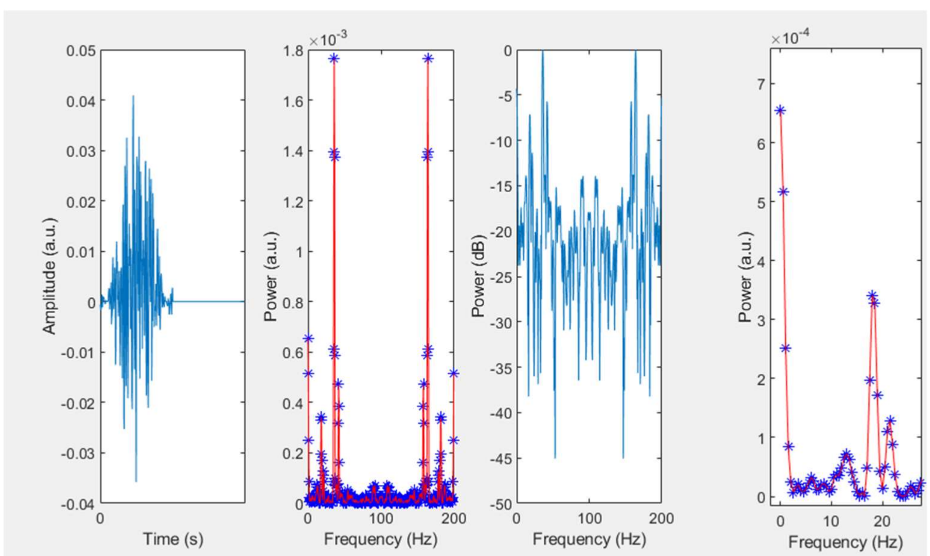

Figure 10: Plot and zoom of the second second using Blackman window at 0.5Hz resolution

C. Discussion

In practice the maximum frequency should be 5 times less than the sampling frequency, so here it would have been 40Hz. However, since the given signal is unknown, we cannot know the recording condition. For instance, we could already have a filter, or on the contrary, some noisy signals can be added. This is why we should stay at the theoretical stage to study this signal.

Also, by applying windows, this causes its Fourier transforms to develop non-zero values after the maximum frequency. So it will make the Nyquist-Shannon theorem to be respected.

We should also bear in mind that applying a window is altering the signal in a certain way.

Furthermore, adding a series of 0 at the end of our signal is, again, not ideal and could distort the given signal. So applying a window and analysing at a very good resolution should be taken into account during the analysis.

In addition, the result section talks about frequencies that are in our range of interest of [5; 20]Hz but other peaks such as 36Hz for example, could have been provided in the result section, but they were less relevant.

Lastly, on every Figures, we can see peaks at 0Hz, which, in fact, is the result of some hidden signals that may be considered as noise. The magnitude of this peak should be the average of the overall signal

Overall, this Signal Processing midterm aimed to analyse an unknown signal in order to extract some frequencies that are inside. To do so, the split of a signal could be done to be more precise due to non-stationarity everywhere. And it also aimed to know how to explain the result obtained after applying a rectangular and blackman window at different resolution levels.

To finish, I think there are frequencies at 6 - 6.5 – 13 – 18 and 18.5 Hz in our range of interest.